Effect of friction on uniaxial compression of bread dough

M. N. CHARALAMBIDES, S. M. GOH, L. WANIGASOORIYA, J. G. WILLIAMS, W. XIAO
Imperial College London, Mechanical Engineering Department, South Kensington Campus, London SW7 2AZ

The mechanical and frictional properties of foods have received considerable attention in recent years as accurate material data are needed for the simulation of large scale processing operations as well as providing a basis for product quality and development. In this study, uniaxial compression tests were performed on bread flour/water dough. Cylindrical samples of various heights and diameters were tested under both lubricated and non lubricated conditions. The stress-strain data showed a dependency on sample dimensions when no lubricant was used due to surface friction effects. This dependency was eliminated when silicon oil of 500 centistokes viscosity was used as a lubricant and the true stress-strain data were obtained experimentally. A theoretical expression derived from an equilibrium analysis of a compressed disc under frictional conditions was used to determine the coefficient of friction and the true stress-strain curve from unlubricated data. The true stress-strain data were also determined by an empirical extrapolation procedure. Finally, an iterative numerical procedure based on Finite Element Analysis confirmed the calculated values for both the coefficient friction and the true stress-strain curve.

1. Introduction

The uniaxial compression test is the most popular means of deriving the stress-strain properties of soft foods and of biological materials in general. An alternative, the uniaxial tension test, is used less frequently because of the problems related with the gripping of soft samples. The drawback of the compression test however, is that the friction between the sample and the loading platens leads to inhomogeneous deformation. Specifically, the material next to the platens is restrained from radial movement and the material appears to be stiffer than it truly is. This leads to 'apparent' as opposed to 'true' stress-strain calculations, when the usual expressions for stress and strain and the experimental load-deflection data are used. As the effect of friction is confined at the two regions adjacent to the platens, a dependence of 'apparent' stress-strain data on sample dimensions is observed. For cylindrical samples, the stress-strain curve will appear to be higher for smaller height/diameter ratio because the volume of the affected material is a larger proportion of the total sample volume.

In an earlier study, the authors reported data obtained from uniaxial compression of two cheeses [1]. Lubricated tests as well as non-lubricated tests were performed. Methods to derive the true stress-strain curve and the coefficient of friction from the non-lubricated data were then derived. In the current paper, the validity of these methods is tested further by applying them to bread dough, a material which is significantly softer than the cheeses of the previous study [1]. Producing accurate samples of dough and material characterisation in general is more difficult. Therefore, this new study will highlight any limitations of the suggested methods for dealing with friction in compression tests and necessary refinements to these methods will be identified.

2. Experiments

Simple flour/water dough was mixed using a laboratory 6-pin mixer. The flour was supplied by General Mills with a blend composition of: 13.25% ± 0.75% moisture, 10.5% ± 0.35% protein and 0.5% ± 0.03% ash contents. Ash and protein are both quoted on the 14% moisture basis standard. The dough was made by mixing 198.5 g of flour with 120 g distilled water and 1.5 g salt (sodium chloride), giving a total of 320 g of dough from each mix. All samples were mixed for three minutes at a constant speed in ambient conditions.

In order to study the effect of friction, samples of various dimensions were produced using cylindrical moulds made from polutetrafluorethylene (PTFE). Each mould was placed on a square PTFE sheet (bottom plate) coated with paraffin oil, as shown in Fig. 1. A strip of non-stick greaseproof paper was used to line the inside of the cylindrical moulds. The dough was pressed into the moulds by hand and any excess material was scraped off with a sharp edge. Another PTFE plate (top plate) coated with paraffin oil was then positioned on top of the filled mould and a small weight was left on top

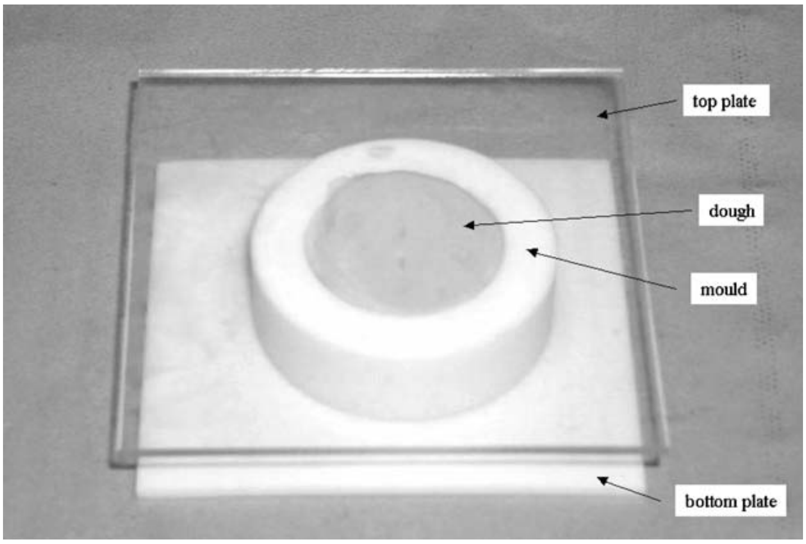

Figure 1 Specimen preparation assembly. A PMMA top plate is shown here for clarity purposes, in real tests both top and bottom platens were made from PTFE.

of the plate for 10–15 min. The top plate was then removed using a sliding motion, the mould was lifted off and the sample was left to relax for 45 min. The sample retained its shape during this time as the greaseproof paper provided the necessary support. The sample was then transported to the loading platens using the bottom plate. Prior to testing, the bottom plate was removed using a sliding motion. Finally the greaseproof paper was peeled off. The paraffin oil which was used to avoid moisture loss from the sample was carefully removed using absorbent paper. This rather tedious preparation procedure ensured samples had the correct cylindrical geometry and reduced the scatter in the experimental data.

The following mould dimensions were used:

(i) diameter of 40 mm and heights of 6, 10, 16 and 20 mm,
(ii) diameter of 20 mm and heights of 5, 10 and 20 mm.

The 40 mm diameter samples were easier to produce. The 20 mm diameter samples were more difficult to extract from the moulds without damaging them. However, less mass of dough is needed to produce the 20 mm diameter samples. The exact height of each sample was recorded within ± 0.5 mm prior to testing. For each height, a minimum of four replicate samples were tested.

The experiments were performed at room temperature at a constant strain rate of 40 min^{-1}. To keep the strain rate constant, the crosshead speed was set to decrease exponentially with time. The testing machine used was an INSTRON model 5543 which has this optional feature. Two frictional conditions were examined by performing two series of tests for each sample diameter. Firstly, tests were performed where no lubricant was applied to the loading platen—sample interface before testing. In this case, the loading platens were made of polymethylmethacrylate (PMMA). Secondly, silicon oil of 500 centistokes viscosity was applied to the loading platens which were made of PTFE.

Corresponding values of load, P, and deflection, δ, were recorded on a PC connected to the testing machine. These were used to calculate the mean stress, p, and strain, ε, from:

$$p = \frac{Ph}{\pi R^2 H} \tag{1}$$

and

$$\varepsilon = -\ln \frac{h}{H} \tag{2}$$

where H is the original height, R is the original radius and h is the current height ($= H - \delta$). Note that Equation 1 assumes a constant volume deformation and corrects the stress for the changes in sample dimensions during compression. The strain as defined in Equation 2 is the Hencky strain and for large deformations is a better estimate of the real strain in the sample than engineering strain. For clarity purposes, all values of stress and strain will be presented as positive, even though they are compressive. These positive values are achieved by introducing the minus sign in Equation 2 and by taking the sign of the load P in Equation 1 as positive.

The stress—strain curves for $D = 40$ mm and $D = 20$ mm when no lubrication was used are shown in Figs 2 and 3 respectively. For each nominal height, an average curve obtained from all samples is shown. The error bars are set equal to ± one standard deviation. The height effect is evident; shorter samples led to a higher stress—strain curve, i.e. the material appears stiffer. The results when silicon oil was used are shown in Figs 4 and 5. The effectiveness of this lubricant in eliminating frictional effects is obvious as all heights now lead to a single stress—strain curve for both diameters. Thus the lubricated test data can be assumed to represent the true stress - strain curve of the material at the specified test conditions.

Photographs of samples during testing with and without lubrication are shown in Fig. 6. The samples shown

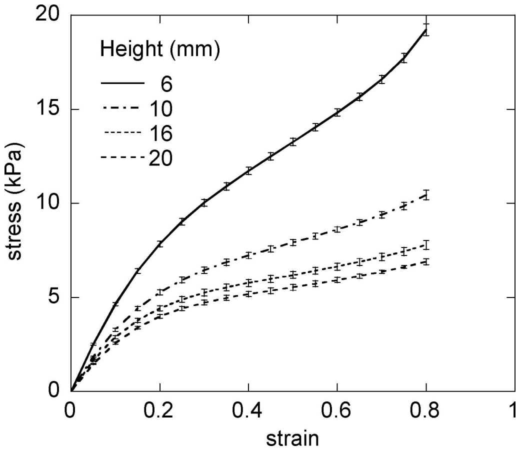

Figure 2 Stress-strain curves from samples of 40 mm diameter and varying height, without lubrication.

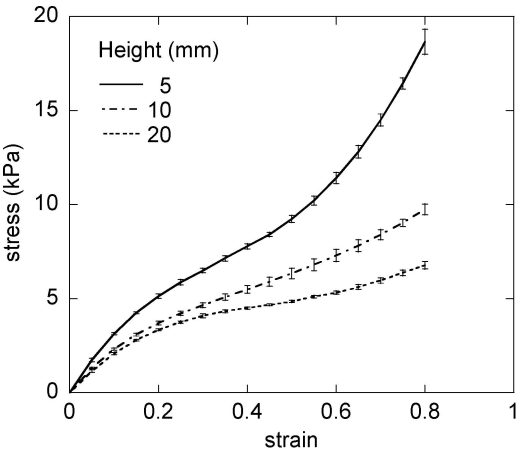

Figure 3 Stress-strain curves from samples of 20 mm diameter and varying height, without lubrication.

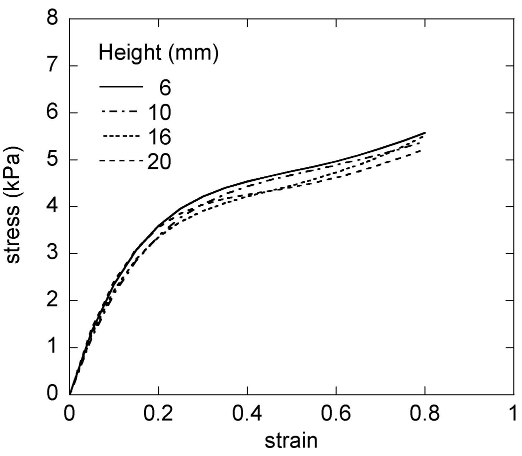

Figure 4 Stress-strain curves from samples of 40 mm diameter and varying height, with lubrication. For clarity purposes, error bars are omitted.

had an initial height of 20 mm and a diameter of 40 mm. The applied strain is approximately 0.6. The barrelling is more pronounced in the photograph of the unlubricated test; this is further evidence that the chosen lubricant was effective.

The averages of the lubricated curves are compared in Fig. 7. The two curves are close to each other, with the 20 mm curve being somewhat lower, particularly at larger strains.

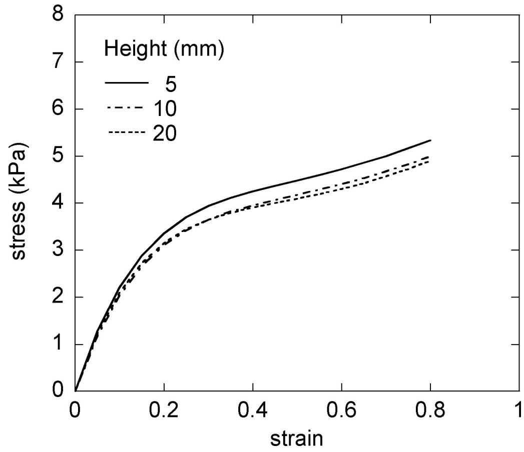

Figure 5 Stress-strain curves from samples of 20 mm diameter and varying height, with lubrication. For clarity purposes, error bars are omitted.

3. Analytical solution

In this study, it was possible to eliminate frictional effects by using silicon oil as a lubricant. However, there might be cases for which friction cannot be eliminated completely. The data derived from the compression experiments would then be influenced by friction. A method which enables the calculation of the true stress-strain curve from such data would therefore be useful. In addition, the coefficient of friction is an important property of the contact surface that will be needed for modelling of industrial processes such as rolling, extrusion or cutting. Methods for deriving the true stress-strain curve as well as the coefficient of friction were derived in our previous work on cheeses [1] and they are briefly described here.

Considerations of equilibrium on a section of a compressed disk under friction leads to the following expression for apparent stress, p, as a function of the 'true' stress σ_0, original height, H, original radius, R, coefficient of friction, μ, and applied strain, ε, [2]:

$$p = \frac{\sigma_0}{2}\left(\frac{H}{\mu R}e^{-\frac{3}{2}\varepsilon}\right)^2\left[e^{\frac{2\mu R}{H}e^{\frac{3}{2}\varepsilon}} - \frac{2\mu R}{H}e^{\frac{3}{2}\varepsilon} - 1\right] \quad (3)$$

In deriving Equation 3, the following assumptions were made:

(i) there is no barrelling of the edges of the disk,
(ii) the thickness of the disk is small enough so that the axial compressive stress is constant through the thickness.

Both of the above assumptions are strictly not true as without a lubricant, barrelling followed by folding of the sample is observed. This would imply that the axial stresses are not uniform. Nevertheless, these assumptions are necessary in order to simplify the problem and therefore derive an analytical solution. The experimental data were fitted to Equation 3 using the Solver function of the Microsoft Excel software. In order to facilitate the approximation, the experimental data are rearranged such that corresponding mean stress p and sample height H data are available for a constant value of strain ε. The results for the 'true' stress-strain

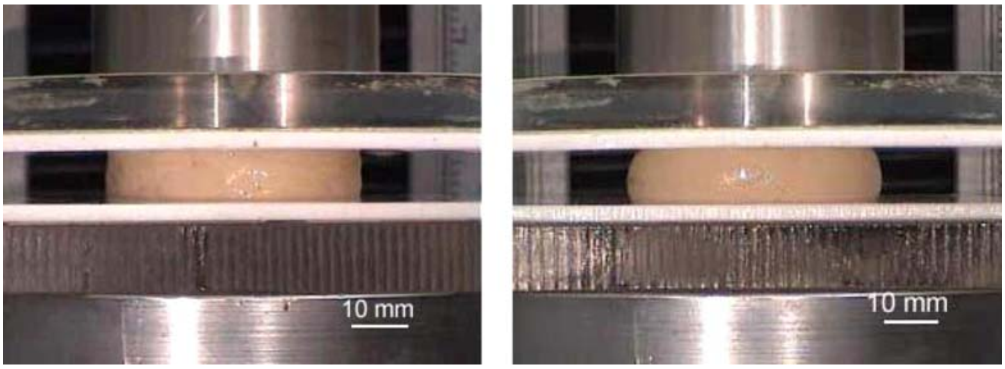

Figure 6 Deformed samples under lubricated and unlubricated conditions.

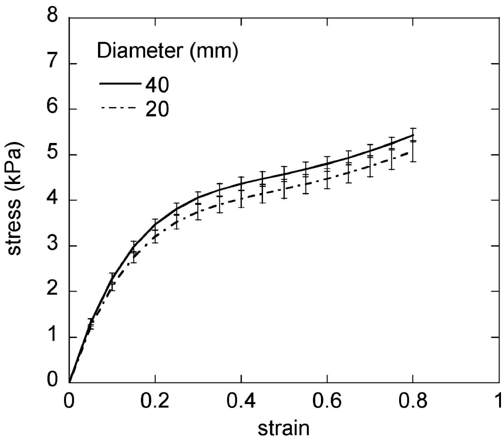

Figure 7 Comparison of lubricated stress-strain curves from samples of two diameters.

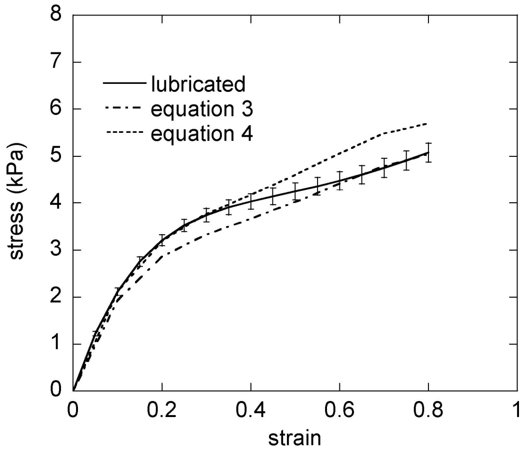

Figure 9 Comparison between experimental lubricated and calculated stress-strain curves using data from 20 mm diameter samples.

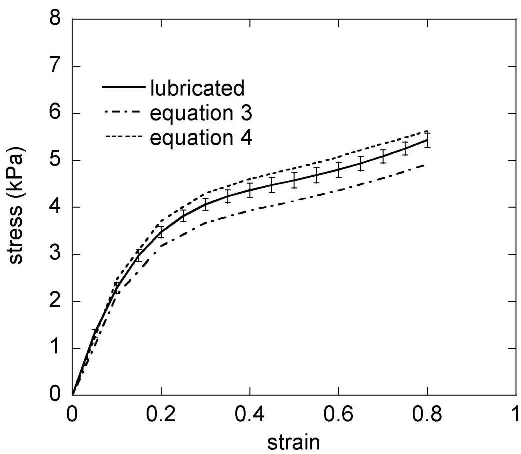

Figure 8 Comparison between experimental lubricated and calculated stress-strain curves using data from 40 mm diameter samples.

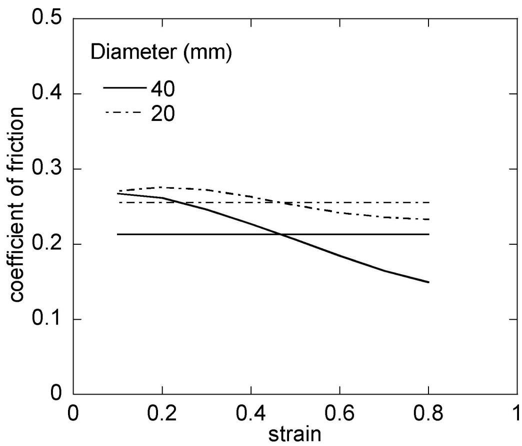

Figure 10 Coefficient of friction versus strain corresponding to the two sample diameters. Horizontal lines represent the average of the data.

curve, i.e. σ_0 versus ε, are shown in Figs 8 and 9 for $D = 40$ mm and $D = 20$ mm respectively. The lubricated stress-strain curve is also shown for comparison purposes. The prediction from Equation 3 is seen to be reasonable for both diameters, as the calculated data are close to the lubricated curves. The coefficient of friction is also calculated using Equation 3 and the results from both sets of data are shown in Fig. 10. It is observed that μ varies with strain, the variation being larger for the larger diameter data. This is probably due to the assumptions made in the analysis; the 40 mm diameter samples have smaller sample height to sample diameter ratios (H/D) therefore the effect of barrelling and of the non-uniform axial stress distribution is larger. Nevertheless, the average values of μ for $D = 40$ mm and $D = 20$ mm are 0.21 and 0.26 respectively which are in good agreement.

In the previous work [1] it was shown that the alternative empirical relationship shown below gave more accurate results than Equation 3 for the true stress-strain curve:

$$p = \sigma_0 + B\left(\frac{1}{H}\right)^2 \qquad (4)$$

where B is a constant. Equation 4, which does not allow estimation of μ, was used to approximate the experimental data and the results are also shown in Figs 8 and 9. The predictions from this empirical relationship are found to be of similar accuracy as Equation 3, in disagreement with our previous work on cheeses [1] where Equation 4 was found to be more accurate than Equation 3.

4. Numerical modelling

The ABAQUS commercial finite element software package [3] was used to model the compression of a cylindrical sample between two flat, rigid platens. The model is axisymmetric and includes the top half of the cylinder only, since the middle surface is a plane of symmetry. Four noded quadrilateral elements were used. A reference node on the rigid surface was displaced in the vertical direction such that the maximum imposed strains matched those measured in the experiments. The material was assumed to behave as a linear elastic—plastic material. The onset of plasticity, usually defined by the point of non-linearity on the stress-strain curves, was taken to be at 2.5% strain. Work hardening was used to accommodate the rise in stress beyond the point of first yield. Incompressible behaviour i.e. a Poisson's ratio of 0.5 was assumed. A value of 0.499 was used in the finite element model instead to avoid possible numerical problems associated with truly incompressible behaviour. The classical isotropic Coulomb friction model available in the software was used. This defines the critical shear stress at which sliding of the surfaces starts as a fraction of the contact pressure, the fraction being equal to μ.

4.1. Verification of Equation 3

In order to check the analytical predictions of Equation 3, the calculated stress-strain curve and the average coefficient of friction were entered in the Finite Element program. The numerical load-deflection diagrams can then be compared to the experimental data from the unlubricated tests; a good agreement would imply that Equation 3 was sufficiently accurate.

The 40 mm diameter data were used such that the stress-strain curve used in the numerical simulation is the one shown in Fig. 8 (Equation 3) and the coefficient of friction is 0.21. The resulting comparison between the numerical and experimental load-deflection data is shown in Fig. 11 for all sample heights. The observed agreement verifies the ability of Equation 3 to model the unlubricated compression of dough cylinders.

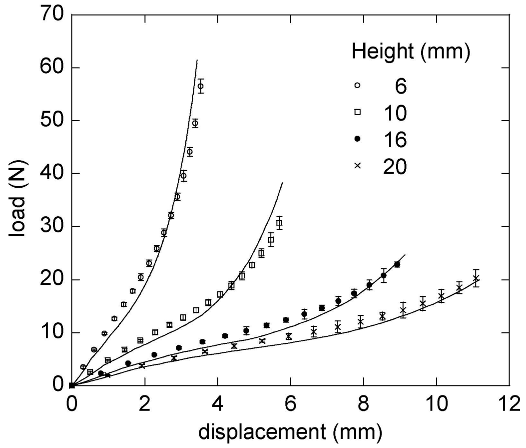

Figure 11 Comparison of numerical and experimental load-displacement plots for $D = 40$ mm. Points and lines represent experimental and numerical data resprectively.

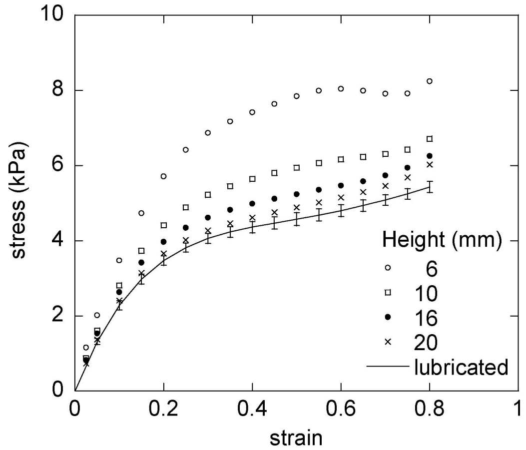

Figure 12 Comparison between the stress-strain curves predicted from the iterative numerical procedure and the lubricated curve when μ was set to 0.1.

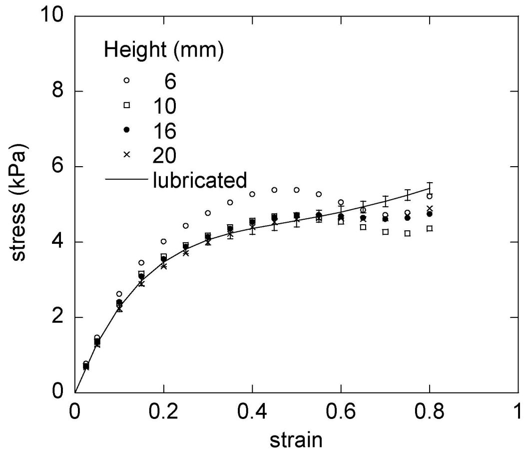

Figure 13 Comparison between the stress-strain curves predicted from the iterative numerical procedure and the lubricated curve when μ was set to 0.2.

4.2. Iterative analysis

A procedure for determining the stress-strain curve from unlubricated test data via an iterative finite element analysis was originally proposed by Parteder and Bunten [4] in a study of compression of steel tested at 1280°C. The same procedure was successfully used in a previous study on cheese [1]. In both of the mentioned studies, it was assumed that the coefficient of friction is known such that solutions of 'true' stress-strain only are sought. Here, we will attempt to determine both the 'true' stress-strain curve as well as μ from the unlubricated data.

The iterative procedure is as follows. For each sample height, the stress-strain curve calculated from the experimental load-deflection data is used as a first estimate of the true stress-strain curve. Five values of

μ were assumed, 0.1, 0.2, 0.25 0.3 and 0.4, such that the iterative procedure was repeated five times for each height. In this way, a numerical force-deflection curve or force—global strain curve $F_{\mathrm{num}}(\varepsilon)$ was obtained for each value of μ. Comparison between the experimental curve $F_{\mathrm{exp}}(\varepsilon)$ and the numerical curve $F_{\mathrm{num}}(\varepsilon)$ leads to the estimation of the correction factor as a function of strain, $c(\varepsilon)$, i.e.:

$$c_{\mathrm{i}}(\varepsilon) = 1 - \frac{F_{\mathrm{exp}}(\varepsilon)}{F_{\mathrm{i\ num}}(\varepsilon)} \quad \text{for } i = 0, 1, 2 \ldots \qquad (5)$$

where i is the number of iterations. The corrected stress-strain curve is then calculated from:

$$\sigma_{\mathrm{i+1}}(\varepsilon) = \frac{\sigma_{\mathrm{i}}(\varepsilon)}{1 + c_{\mathrm{i}}(\varepsilon)} \qquad (6)$$

This iterative scheme was used with the data corresponding to the 40 mm diameter. It was found that typically two to four iterations were needed to bring $c(\varepsilon)$ to almost zero, with shorter specimens requiring a larger number of iterations. This is in contrast to previous work on cheese where a single iteration was sufficient for all sample heights [1, 4]. The reason for the larger number of iterations in dough is probably due to the different shape of the stress-strain curve which could slow down the convergence rate of the numerical method.

The corrected stress-strain curves for all heights and corresponding to the five values of μ are shown in Figs 12–16. The curve obtained from the lubricated experiments is also shown for comparison purposes. It is observed that for the case of $\mu = 0.25$, the curves corresponding to the various sample heights are closest to each other as well as to the lubricated curve. For the other four values of μ, the iterated curves do not agree with each other as well, nor with the lubricated curve. The disagreement between the various curves increases as μ is further away from the value of 0.25. This implies that the correct value of μ is close to 0.25. This agrees well with the average value obtained from Equation 3 using the smaller, 20 mm diameter, data which was 0.26 (see Fig. 10). The agreement with the average value from the 40 mm diameter data, i.e. 0.21, is less good. This could be explained by the fact that the ratio (H/D) varied from 0.15 to 0.5 for the 40 mm samples and from 0.25 to 1.0 for the 20 mm samples. The smaller ratios for the 40 mm samples would mean that the assumptions made in deriving Equation 3 are less accurate. Therefore the values of μ calculated using the 40 mm data would be less accurate than those obtained from the 20 mm data. This is also highlighted by the larger variation in μ for the 40 mm data, as shown in Fig. 10.

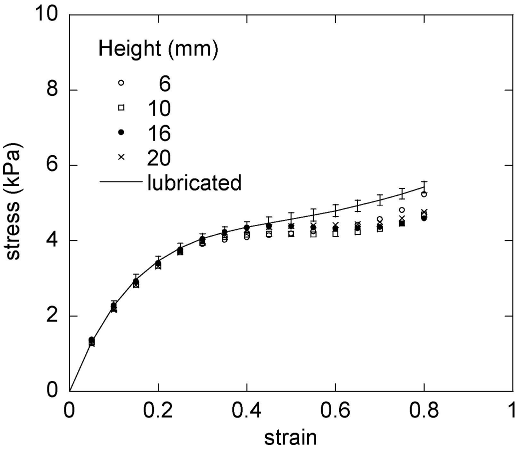

Figure 14 Comparison between the stress-strain curves predicted from the iterative numerical procedure and the lubricated curve when μ was set to 0.25.

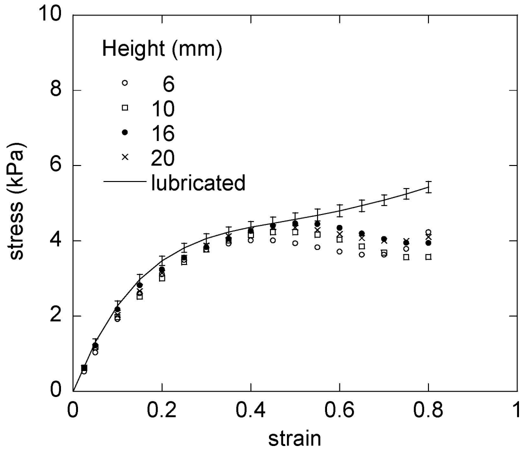

Figure 15 Comparison between the stress-strain curves predicted from the iterative numerical procedure and the lubricated curve when μ was set to 0.3.

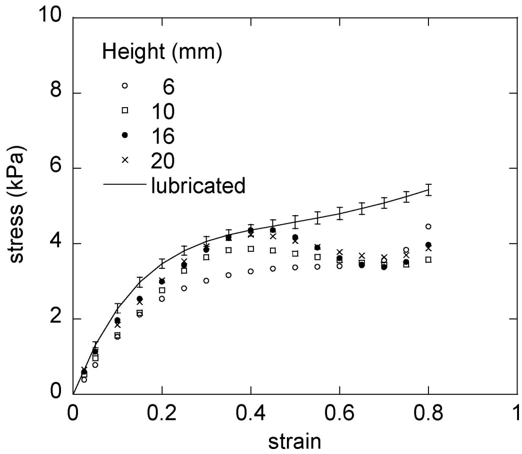

Figure 16 Comparison between the stress-strain curves predicted from the iterative numerical procedure and the lubricated curve when μ was set to 0.4.

5. Conclusions

Silicon oil with a viscosity of 500 centistokes was found to eliminate friction in uniaxial compression tests of bread dough. The evidence for this is that there was no effect of sample geometry on the test data when this lubricant was used.

Two expressions were used to determine the 'true' stress-strain curve from unlubricated data. One was based on equilibrium analysis of a compressed disk under frictional conditions (Equation 3) and the other was an empirical expression (Equation 4). The latter is essentially an extrapolation of the apparent stress data to a zero $(1/H)^2$ value, H being the original sample height. It was found that the predictions for the 'true' stress-strain curve from both equations were good. Equation 3 also allows the estimation of μ; the average values of 0.21 and 0.26 were derived for $D = 40$ mm and $D = 20$ mm respectively.

Equation 3 was further verified by entering its predictions for stress-strain and average coefficient of friction to a finite element simulation of the unlubricated test; the resulting load-displacement data agreed well with the experimental data. An iterative finite element analysis procedure was also used to derive the 'true' stress-strain curve and μ simultaneously from unlubricated test data. It was found that the predicted stress-strain curves were very close to each other as well as to the curve measured from the lubricated experiments when μ was close to 0.25. This is in good agreement with the predictions from theory (Equation 3).

Acknowledgements
The authors would like to acknowledge the financial support of General Mills, USA. They would also like to thank Dr S. Chakrabarti of General Mills for the useful discussions and suggestions.

References
1. M. N. CHARALAMBIDES, S. M. GOH, S. L. LIM and J. G. WILLIAMS, *J. Mater. Sci.* **36** (2001) 2313.
2. G. DIETER, "Mechanical Metallurgy", 3rd ed. (McGraw Hill Book Company, New York, 1986), p. 539.
3. ABAQUS User's Mannual, Version 6.4, Hibbit Karlsson and Sorensen, USA, 2003.
4. E. PARTEDER and R. BUNTEN, *J. Mater. Process. Technol.* **74** (1998) 227.

Received 19 November 2004
and accepted 14 January 2005